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Full Waveform Inversion (FWI) in 3D Synthetic Geological Modelling

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Abstract - Modeling the Earth's subsurface is important in geophysics, for example, tomography and Full Waveform Inversion (FWI). Especially for FWI, it is a method in the geophysical field that is used to describe the subsurface with a high-resolution image by utilizing all parts of observation data (seismic traces) based on a velocity model. In this paper, the 3D synthetic geological modelling was generated as an initial model; then, based on it, seismic traces and also shot gathers were simulated by arranging sources and geophones, and finally, the FWI was done. After seismic traces (assumed as acoustic wave equations), complete with the smoothness, were obtained, then a hyperbolic partial differential equation between the time and space domains was applied before entering the inversion process. During the inversion process, the final seismic traces were obtained by velocity updating in the entire space and time until they reached zero misfit, so wave propagation and the final velocity model could be observed. The pattern changes of seismic traces, shot gather, wave propagation, and FWI could be described by modifying the seismic source and geophone location in the geological model. However, by generating a 3D synthetic geological model, there is much flexibility for getting various descriptions of acoustic wave simulations.

Keywords: 3D geological modeling, full waveform inversion, wave equation, partial differential equation

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Introduction

Full Waveform Inversion (FWI) is a seismic imaging tool with high resolution. It is similar to seismic tomography. Both of them need velocity as the initial model, then by generating ray seismic between the source and the geophone, seismic tomography updates the velocity model using an inversion technique. However, FWI utilizes all parameters, such as seismic trace (as wave equation) and its phase, by applying the finite difference method and updating its velocity model during the inversion. It is always used in seismic exploration to describe the wave propagation from a source to geophones in a geological model (Almuteri and Innanen, 2016; Virieux and Operto, 2009). Based on the explanation and according to Operto *et al*. (2023), FWI is included in the nonlinear inversion case, because it considers the oscillation process in The Wave Equation.

In a numerical problem, the wave equation is included in a hyperbolic partial differential equation. As geophysicists, it is well known that this problem can be solved by using Taylor Expansion. Considering the natural wave characteristic expressed in Fermat's and Huygens' Laws, wave oscillation is influenced by the medium, represented by velocity. In other words, FWI tries to update the velocity of the medium by utilizing the wave equation (Carneiro *et al*., 2014). So, the velocity becomes the input data in FWI . Seismic waves can be reflected, refracted, and propagate in any direction, depending on the heterogeneity of the medium. It will be depicted in a seismic trace as the data. Then, similar to the other cases in geophysics, the data is inverted to get the FWI model by using the least square method.

However, the previous explanation explains that the medium and its geometry are very important in FWI. In this paper, wave propagation was tried to be visualized, and the FWI in the synthetic 3D Geological Modelling was done. 3D Geological Modelling is more complex and realistic than usual 2D Geological Modelling. It consists of geological and geophysical information describing the 3D geometry (Alvarado-Neves *et al*., 2024; De Donatis *et al*., 2005), and hopefully, the FWI model will have its own characteristic. In other words, a 3D geological modelling will give a different FWI pattern from others. Besides that, geological modelling sometimes needs computerization to make the visualization easier. Some examples of 3D Geological Modelling are to describe solid minerals, oil and gas, urban planning, and geological hazard fields. There is also integration of geological modelling and time domain (Cao *et al*., 2024).

There are so many software programmes that can be used for 3D geological modelling. However, in this paper, the 3D synthetic geological modelling would be made by using Gempy (de la Varga *et al*., 2019), a Python module. Gempy allows users to make a 3D structural geological model based on maps, borehole data, or stratigraphic information. It can also predict the topographic form by a statistical approach. The other advantage of using Gempy is that the input data can be formed in CSV (Jüstel *et al*., 2023). Figure 1 shows the example of 3d Geological Modelling using Gempy, while Figure 2 depicts the FWI example. It can be said that Gempy gives much flexibility in building our own geological model.

Finally, based on FWI forward and inverse modelling, with additional information on wave equations and seismic simulations, such as source and geophone location, synthetic shot gathers and wave propagation can also be made. Figure 3 shows both of them. The wave propagation is coming from the partial derivative itself, while the shot gather model is calculated from the reflectivity and velocity in seismic processing based on 3D Geological Modelling.

Figure 3 (b), from left to right, represents wave propagation over time. It is a wave propagation model. To achieve this, it is important to generate an equation that includes both density and velocity. Then, the order of the hyperbolic partial differential equation was determined as the forward modelling (Hardi and Sanny, 2017).

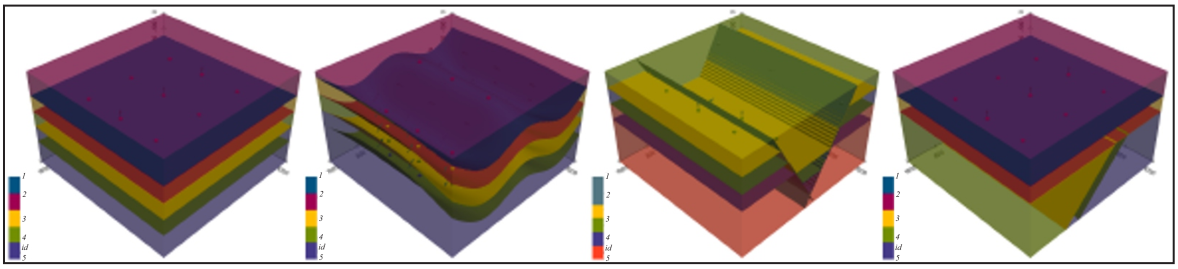

Figure 1. 3D Geological Modeling examples by using Gempy (Jüstel *et al*., 2023).

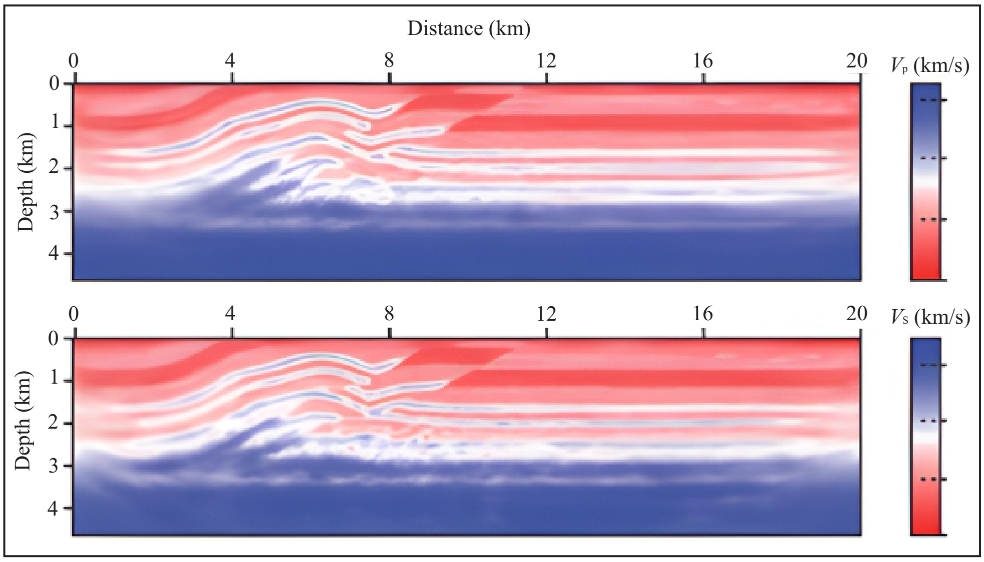

Figure 2. Two examples of FWI based on Vp (top) and Vs (bottom) values of the medium (Virieux and Operto, 2009).

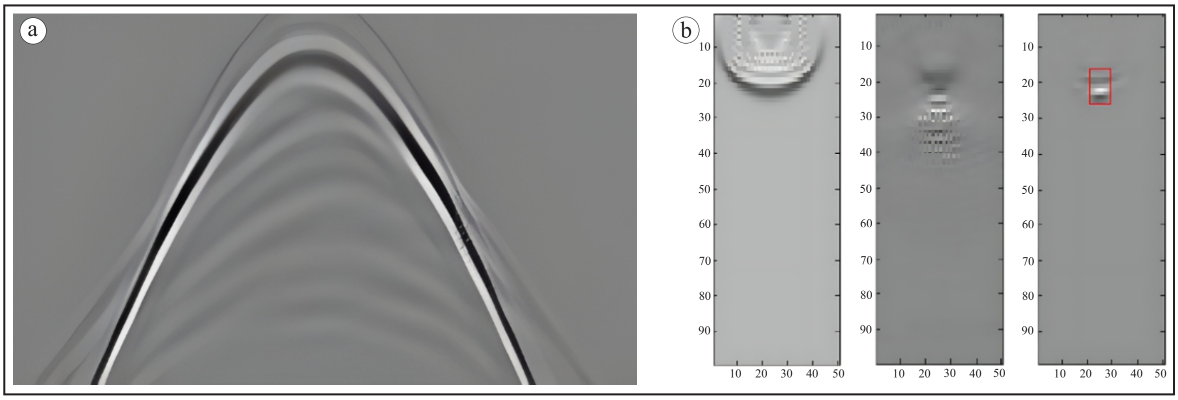

Figure 3. (a) Shot gather example (Yang *et al*., 2020); (b) wave propagation (Hardi and Sanny, 2017).

METHOD

In this paper, a 3D geological model was tried to be built consisting of four layers and two faults in Gempy. Because of our computer capability, a 3D geological model was built as a cube with a side of 1,000 m with 10 m spacing. Coordinates and directions were also set for each layer and the fault. For the velocity value, there were 1.4 km/s, 1.7 km/s, 2 km/s, and 2.3 km/s for all layers, as their lithology. Then nine sources were arranged with 125 m space and 30 m depth, and 100 geophones with 10 m space and 20 m depth. Ricker was used as the mother wavelet with a 0.008 Hz frequency. These arrangements were adjusted depending on the size of the 3D geological cube. All of our processes follow the information of (Caunt, 2021) and Devito (Louboutin *et al*., 2019).

The FWI starts with the wave acoustic equation, which consists of density, source, and velocity parameters that are assumed to be contained in the seismic traces. All of them describe the reflectivity or acoustic impedance in the medium. The wave can be transversed, deflected, or reflected due to the boundary field between two different types of media or heterogeneity. This is a natural characteristic of a wave that can be calculated and modeled using a numerical approach, like in equation (1) below:

$$\frac{1}{V(x)^2}\frac{\partial^2 P(x,t)}{\partial t^2} - \rho(x).\left(\frac{1}{\rho(x)}\nabla P(x,t)\right) = S(x,t) \;....(1)$$

where:

V and ρ are velocity and density in the time (t) and space (x) domain, while

P and S represent pressure and source (Yang *et al.*, 2020).

Both of them influence the attenuation process of the seismic wave, while pressure can change the physical parameters of the medium. Then, if just focusing on velocity changing through the time and space domains, Equation (1) can be written like Equation (2):

$$m\frac{d^2u(t,x,y)}{dt^2} - \Delta u(t,x,y) + \eta\frac{du(t,x,y)}{dt} = q(t,x,y) \text{ ....(2)}$$

where:
m is equal to $\frac{1}{V^2}$
Δ is the Laplace operator, and
η represents damping.

The parameter q is similar to S in Equation (1). There are 1st and 2nd orders of partial differential equations in Equation (2). Based on the numerical approach, each of them can be solved like in Equation (3):

$$\frac{du}{dt} = \frac{u_{t+1} - u_t}{\Delta t} \text{ ....(3a)}$$

$$\frac{d^2u}{dt^2} = \frac{u_{t+1} - 2u_t + u_{t-1}}{\Delta t^2} \text{ ....(3b)}$$

It describes u-shifting in time. It can be connected or combined with the Laplace operator, which consists of a similar pattern in the space domain (Louboutin *et al.*, 2017). Figure 4 shows an illustration of wave propagation in 3D Geological Modelling at two different times based on the Numerical Approach.

The wave equation with a numerical approach as problem-solving is included in Forward Modelling. Then it will be inverted by re-arrange Equation (2) into Equation (4):

$$\frac{\partial q}{\partial x} = \sum_{j=1}^{l}\left[\left(\sum_{i=1}^{r}\frac{\partial F_j(t)}{\partial x_i}\Delta m_i - \varepsilon_j\right).\frac{\partial F_j(t)}{\partial x_i}\right] = 0 \text{ ....(4)}$$

However, Equation (4) can be simplified like in Equation (5)

$$A\Delta m = B \text{ ....(5)}$$

And finally Δx will be observed using the inverse method.

$$\Delta m = A^{-1}B \text{ ....(6)}$$

Notice that in Equation (4), the m_i parameter describes both the space and time domain (Wang *et al.*, 2013). However, Equation (6) is similar to the general inverse method in geophysics. Δx is the parameter model that will be looked for, A is forward modelling, and B represents the data. The best model was observed when the parameter model was really close to the input model, or it can be said the misfit reached zero. Velocity and wave propagation would then be updated during the iteration in the inverse method, both in the space and time domains. Generally, all of the FWI processes can be seen in Figure 5.

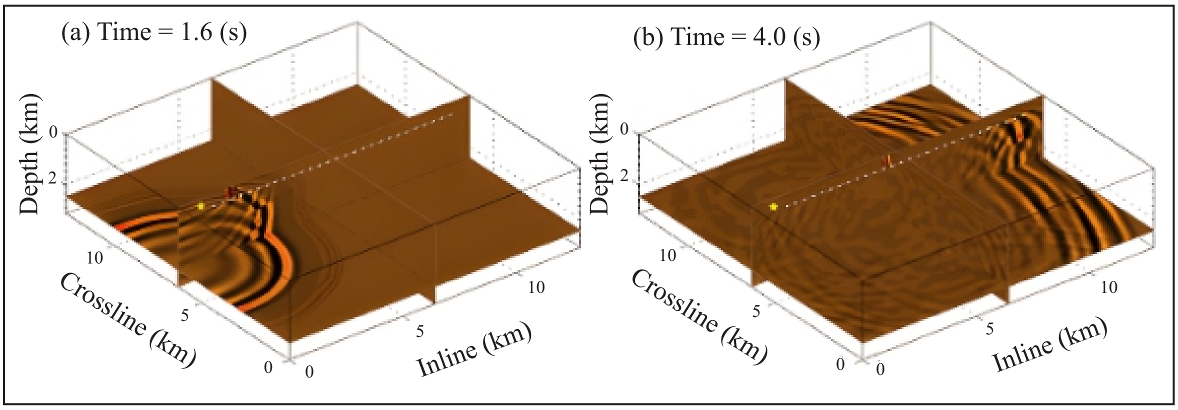

Figure 4. Wave propagation in (a) 1.6 s and (b) 4.0 s (Borisov and Singh, 2015).

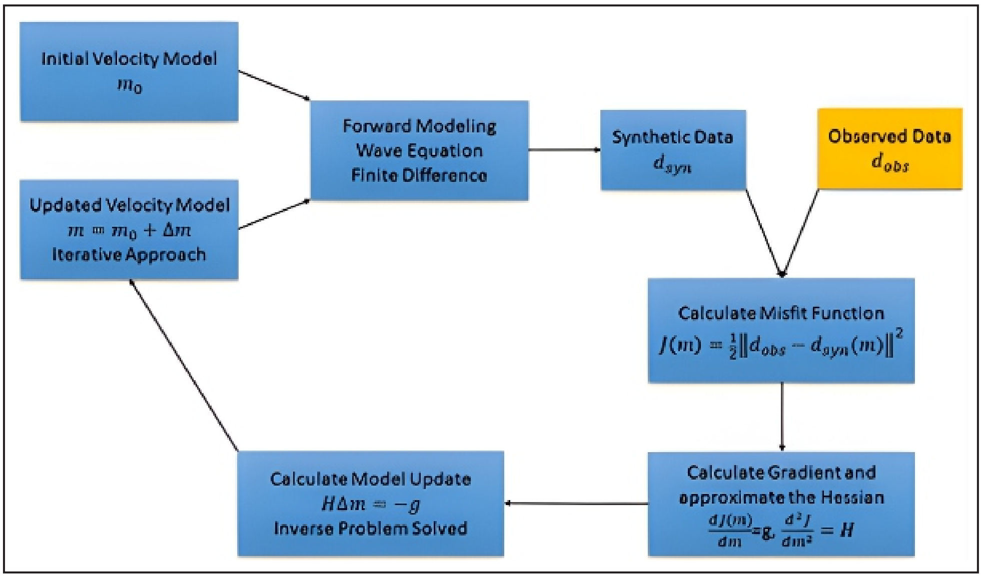

Figure 5. FWI step by step (Akrami, 2017).

RESULT AND DISCUSSION

In this paper, the 3D synthetic geological modelling was tried to be made by adopting a tutorial from Caunt (2021). Four layers and two faults for 3D synthetic geological modelling were made by using the Gempy module from Python, as shown in Figure 6. These layers consist of CO_2, sand, shale, and basement in depth from top to bottom.

The topography in Gempy is built using a statistical approach. As a user, an initial value was just given to determine the azimuth. Based on 3D synthetic geological modelling in Figure 6, this can be visualized in 2D form, like in Figure 7.

The geological model was smoothed first before going to FWI and wave propagation. Then, showing these models was chosen by using the seismic source that was located in the centre of the geological model, as shown in Figure 8. The shot gather shows multiple causes due to the heterogeneity of the four layers. It also represents repeated reflection of the seismic wave and the resulting noise. In the FWI model, the velocity

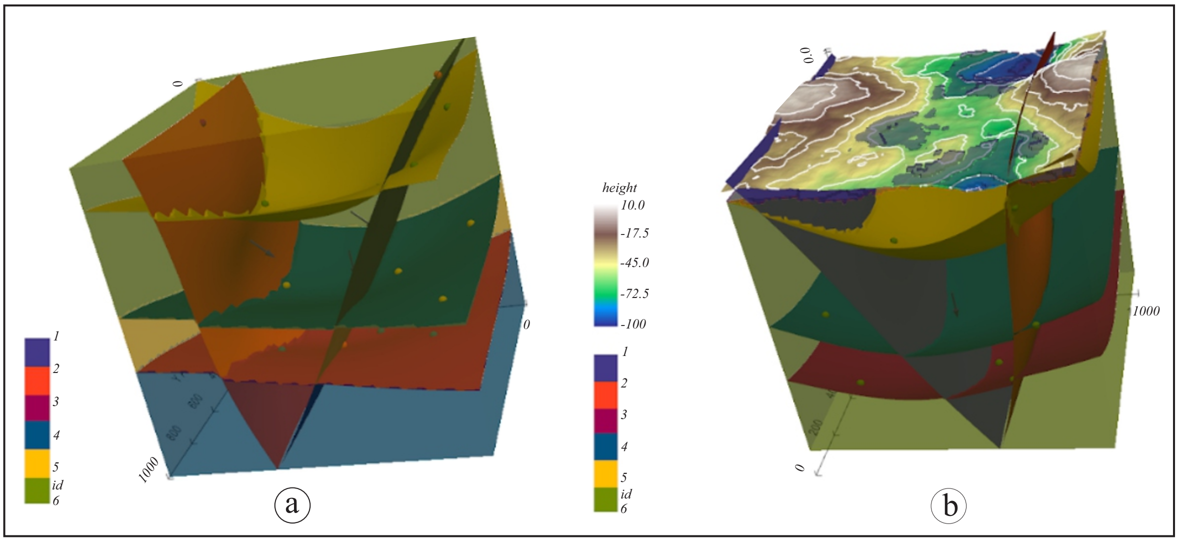

Figure 6. (a). 3D Synthetic Geological Modeling without topography and (b). topography included (Caunt, 2021).

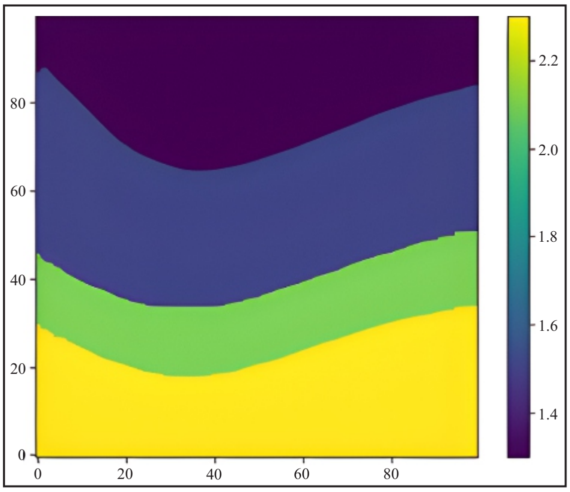

Figure 7. 2D form based on 3D Synthetic Geological Modeling. The colours represent velocity value in each layer.

value becomes greater than the original, caused by updating the velocity during the inversion process. Besides that, the FWI also supports the multiple ripple depiction. Figure 8 (d) shows the differences between FWI and the smooth model. Generally, both of them look similar, but there is slight difference in the bottom part represented by contrast velocity. It might be caused by the smoothness effect, which cannot reach the maximum, resulting in quite high differences in the FWI model.

Sometimes, the inverse model becomes trapped in local minima, including FWI. Ideally, the seismic trace should be seen to determine whether FWI is trapped in local minima or not. If the seismic trace from FWI has a similar pattern to the original, it means that the FWI model is good enough. Unfortunately, seismic traces were not simulated in this paper, so the comparison of the synthetic 1-D velocity profile was tried to be figured out between FWI and the smooth model (Figure 9). There are several synthetic 1-D velocity profiles that are trapped in local minima,

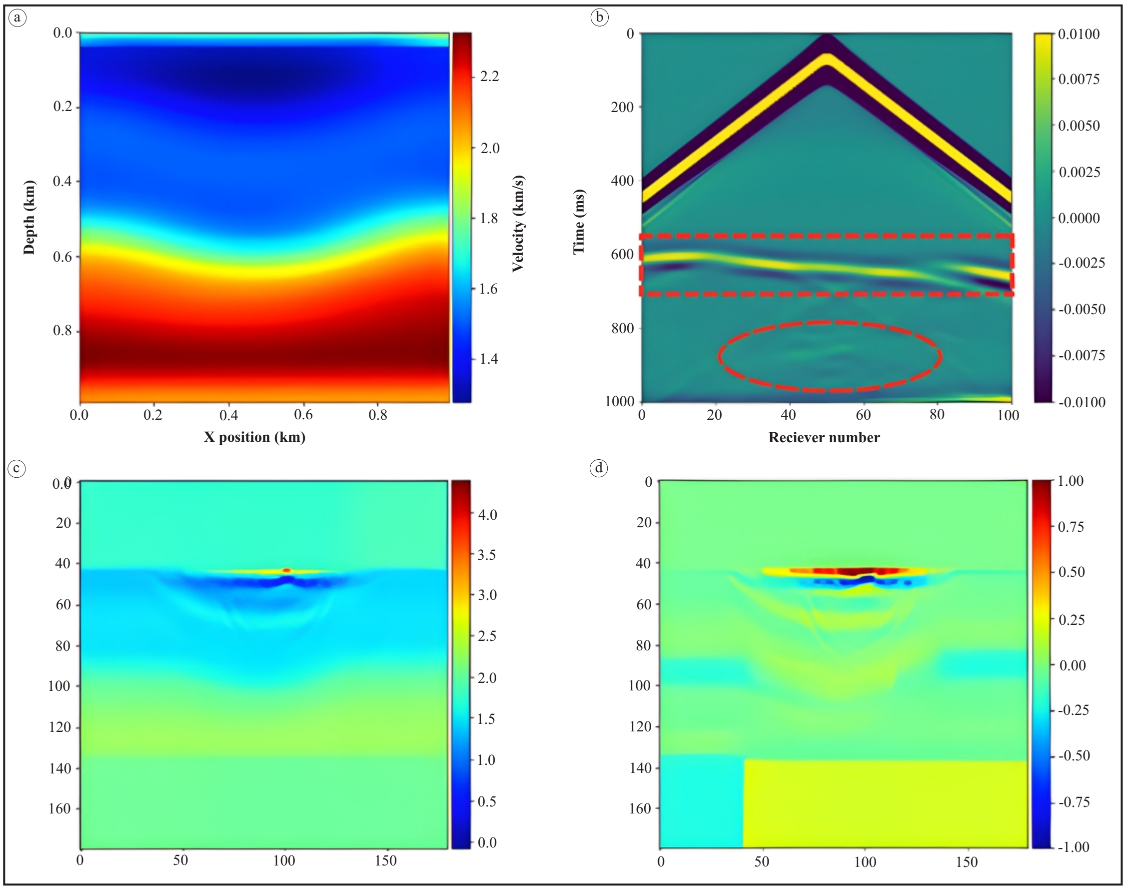

Figure 8. (a) Smoothness of geological model; (b) Shot gather simulation. The colour scale bar represents pick and trough in synthetic seismogram model; (c) Full Waveform Inversion model. The colour scale bar shows its new velocity model; (d) The difference between FWI and geological smooth model.

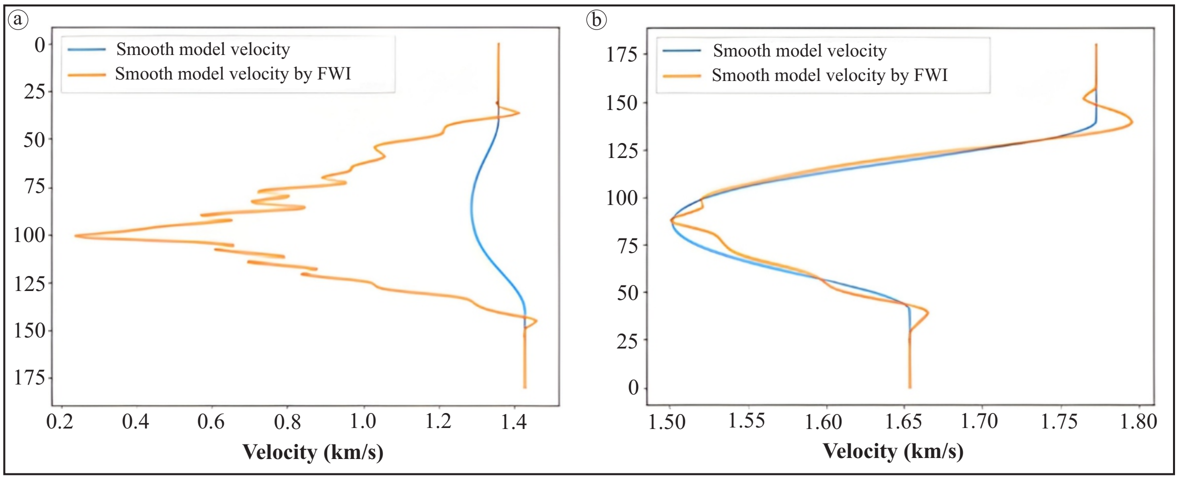

Figure 9. Synthetic 1-D velocity profile from (a) 43rd column and (b) 90th column.

starting from the 43rd column to the 81st column, or about 38% of the geological model. Based on the location, these columns are suitable for the high contrast velocity in Figure 8 (d). It can be said that smoothness is the main factor. Nevertheless, the other 1-D velocity profiles still have high similarity to the original one, which indicates that the FWI model is quite adequate enough.

However, the shot gather model and FWI model in Figure 8 just represent the effect of seismic wave propagation of a seismic source. Similar things will happen to the other seismic source. The layers and multiples are represented by a red dashed box and a red dashed circle. Multiple attenuation is clearly visible here as the effect of the reflected wave caused by the boundary between the two layers. Then, to figure out the wave propagation, a numerical approach was used as a solution of a partial differential equation.

Numerical approaches, especially finite difference methods like in Equations (3.a) and (3.b), update velocity value in time and space domains, covering all space dimensions. It takes a long time to process it manually. Fortunately, Python provides Devito (Witte *et al*., 2020) as a module to generate and simulate wave propagation (Figure 10).

Figure 10 shows wave propagation in 3D synthetic geological modelling using the finite difference method. However, equations (3.a) and (3.b) are the simplest models because they just reach the second partial differential equation. In

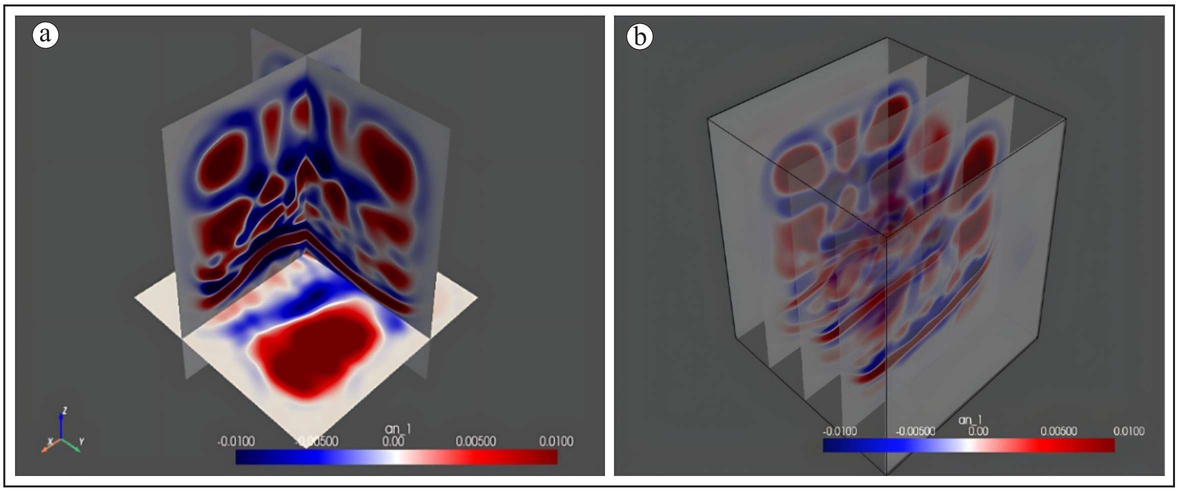

Figure 10. Wave propagation in 3D Synthetic Geological Modeling in (a) x, y, z axis; (b) y axis slices.

Devito, the model is transformed into a complex partial differential equation by applying it across all axes. This model can be different in time, showing that the wave propagates dynamically. A similar pattern is also seen in Figure 10 in 3D modelling. Each slice of the 3D geological volume reflects the seismic wave and might cause multiple reflections.

Conclusion

Subsurface imaging is a main focus in every geophysical field, using both seismic and non-seismic methods. Especially in seismics, describing the subsurface can be done by utilizing seismic waves and an initial model of geological conditions based on several parameters like velocity. Tomography is an example of describing velocity by using information about the ray's path and travel time between sources and geophones. Besides that, there is also Full Waveform Inversion (FWI). Theoretically, FWI uses all of the information from the seismic trace. It can be simulated by generating the geological model, arranging seismic sources and geophone locations, setting the frequency, creating the Ricker mother wavelet, and then inverting the model to obtain the result. Although FWI has a vulnerability to being trapped in local minima, in this paper, it is just about 38% based on a 1-D velocity profile, so the FWI model is still good enough. Finally, a numerical approach was used to describe wave propagation in the whole side of the geological model.

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